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PREDICTION OF WAKE IN A CURVED DUCT

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SUMMARY

Experimental data on the development of an aerofoil wake in a curved stream are compared with calculations based on the k–ε model of turbulence with standard constants and with the model constant C_μ dependent on the local curvature. The mean velocity profile is asymmetric, the half-width of the wake is more on the inner side of the curved duct than on the outer side, and the turbulent shear stress decreases rapidly on the outer side. The standard k–ε model is able to satisfactorily reproduce this behaviour. Making C_μ dependent on the local radius improves the agreement on the inner side but slightly worsens it on the outer side.

KEY WORDS Wake curvature k–ε model of turbulence

INTRODUCTION

A large number of experimental and theoretical investigations have been carried out in the wake region of various bodies in a straight duct. Kovasznay[1] carried out hot wire measurements in the near wake of a circular cylinder, while the far wake was studied by Townsend[2] who introduced self-preservation. According to Ramaprian *et al.*,[3] the near wake is the region $x/\theta < 25$, where x is measured from the trailing edge of the wake-producing body along the streamwise direction and θ is the momentum thickness, and the far wake is defined by $x/\theta > 350$. Sreenivasan and Narasimha[4] have characterized a plane wake by two parameters Δ and W, where $\Delta = b'/(x\theta)^{1/2}$ and $W = (w_0/U_e)(x/\theta)^{1/2}$; b' is the half-wake width, w_0 is the wake defect velocity at the centre of the wake and U_e is the free stream velocity. In the far wake of any body the above two parameters tend asymptotically to constant values which are universal numbers. Wygnanski *et al.*[5] have studied the wakes of various bodies whose momentum thicknesses are identical. They found that the far wake is self-preserving as far as the mean velocity is concerned but lacks the universality of the turbulent structure. They claim that the geometry of the wake generator influences the turbulence structure. Houdeville and Tulapurkara[6] have used the k–ε model to predict the development of the wake with and without a pressure gradient. Tulapurkara[7] has described details of the numerical procedure used to predict these flows. Spalding[8] has developed a general programme for predicting parabolic phenomenon.

Studies of the wake in a curved duct are much fewer in number as compared to those in a straight duct. Savill,[9] Nakayama[10] and Ramjee *et al.*[11,12] have investigated experimentally the

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wake in a curved flow. Pratap and Spalding[13] have investigated the flow in a curved duct of rectangular cross-section using a partially parabolic method. Patankar *et al.*[14] have developed a finite difference procedure to predict the development of turbulent flow in curved pipes using the k–ε model.

Curved ducts are used in many applications such as piping systems, aircraft intakes, refrigeration, air conditioning and centrifugal pumps. Curved flows are characterized by secondary flow effects caused by centrifugal forces acting normal to the flow. Although in the fully developed flow in a curved duct the secondary flow effects would be appreciable, the contamination due to secondary flow may be negligible in the early entry region of a curved duct. When one takes a large-aspect-ratio test section, the effects of secondary flow may be negligible even at a larger distance from the entry section. In this paper the mean velocity profiles and Reynolds shear stress in the curved wake of an aerofoil are calculated using the k–ε model of turbulence and compared with experimental results.

EXPERIMENTAL SET-UP

The experimental set-up is shown in Figure 1(a). Air is blown by a 2 HP motor through a centrifugal blower. To reduce the eddies and vortices and to allow a uniform flow in the test section, a honeycomb, two nylon screens, a settling length and a contraction are provided

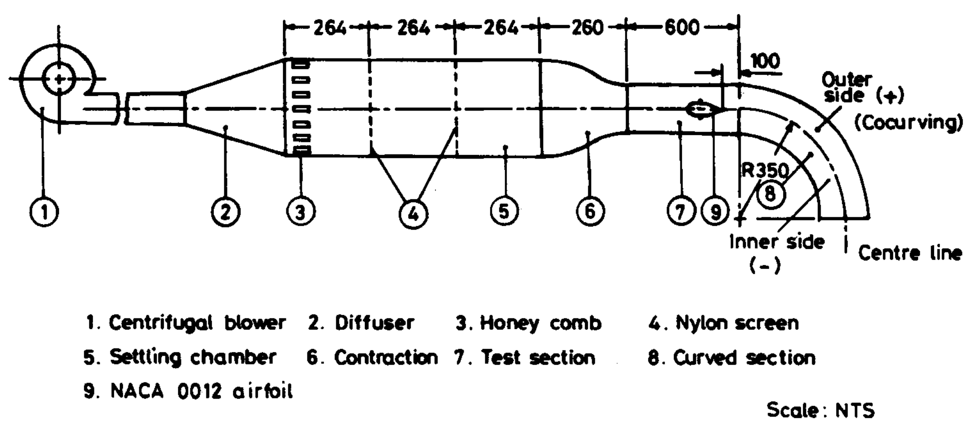

Figure 1(a). Experimental set-up

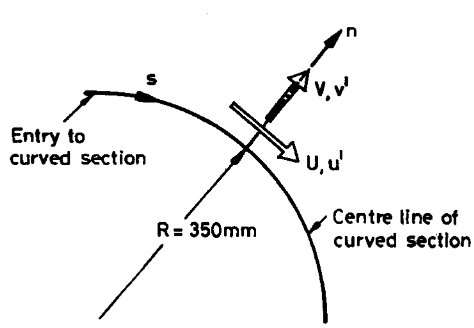

Figure 1(b). The (s, n)-co-ordinate system upstream of the test section. The test section is of square cross-section with a side of 140 mm and the velocity is maintained at a reference value of 15 m s^{-1}.

A symmetric aerofoil (NACA 0012) with a chord length c of 100 mm and a span of 140 mm is kept at zero incidence such that the trailing edge is located 500 mm downstream of the straight test section inlet. Trip wires were placed at $0{\cdot}3\,c$ from the leading edge of the aerofoil so that the boundary layer at the trailing edge is turbulent. A curved duct of the same cross-section as the test section is attached at the end of the test section. The mean radius of curvature of the duct, R, is 350 mm. The wake develops from the aerofoil initially in the straight section and is then subjected to a longitudinally curved flow. Mean velocity and turbulence measurements are taken at various cross-sections from $x/c = 1{\cdot}5$ to $4{\cdot}0$ with single and crossed hot wire probes of 5 μm diameter and a DISA 56C01 constant-temperature anemometer. To check the two-dimensionality of the flow, measurements were taken 35 mm below and above the centreline of the duct at a section where $x/c = 4{\cdot}3$. The mean velocity variations at these locations were within 2% of the centreline velocity distribution.

EXPERIMENTAL RESULTS

The mean velocity U and turbulence quantities were measured at $x/c = 1{\cdot}5$, $2{\cdot}0$, $3{\cdot}0$ and $4{\cdot}0$ with and without the wake-producing aerofoil. In the absence of the aerofoil the mean velocity in the curved section, outside the boundary layers on the duct walls, approximately follows a straight line (chain line in Figure 2). This velocity can be referred to as the potential velocity and denoted by U_p. The quantity U_{Ref} in the figure is the reference velocity in the straight section one chord length ahead of the aerofoil. From the velocity distributions in the presence of the aerofoil (indicated by open circles in Figure 2) it is seen that the wake in the curved flow is not symmetric about the centreline of the duct. The thickness of the shear layer on the inner side is more than that on the outer side; the inner side of the flow here refers to the flow between the centreline of the duct and the inner wall, while the outer side of the flow is between the centreline and the outer wall of the duct.

The velocity defect is obtained as follows. The velocity defect w at a radial station can be defined as the difference between the velocities at that point without the wake and with the wake, i.e. $w = U_p - U$. Figure 3 shows variations of U/U_p with y at different locations. It is seen that the maximum velocity defect w_0 occurs near the centreline of the duct. Let the half-width b' be the value of y at which the velocity defect equals half of its maximum value. It is observed that b' is not same on the two sides of the wake. Its values are shown in Figure 4. The variations of the maximum velocity defect w_0 and an average of the values of the half-widths on the inner and outer sides, b'_{avg}, are plotted in Figure 5. The variation of the turbulent shear stress $\overline{u'v'}$ is shown in Figure 6. It shows that the shear stress distribution becomes more and more unsymmetric with x and the magnitude of the shear stress on the outer side keeps on decreasing. This is perhaps because the velocity gradient $\partial U/\partial y$ is lower on the outer side. Prediction of this behaviour of $\overline{u'v'}$ is an important test for the model of turbulence and is taken up next.

COMPUTATIONAL TECHNIQUE

Houdeville and Tulapurkara[6] found that the k–ε model of turbulence satisfactorily predicts the near-wake regions of symmetric and unsymmetric straight wakes. Prediction with the same model is attempted in the present case of the wake developing in a curved duct. Following Rodi

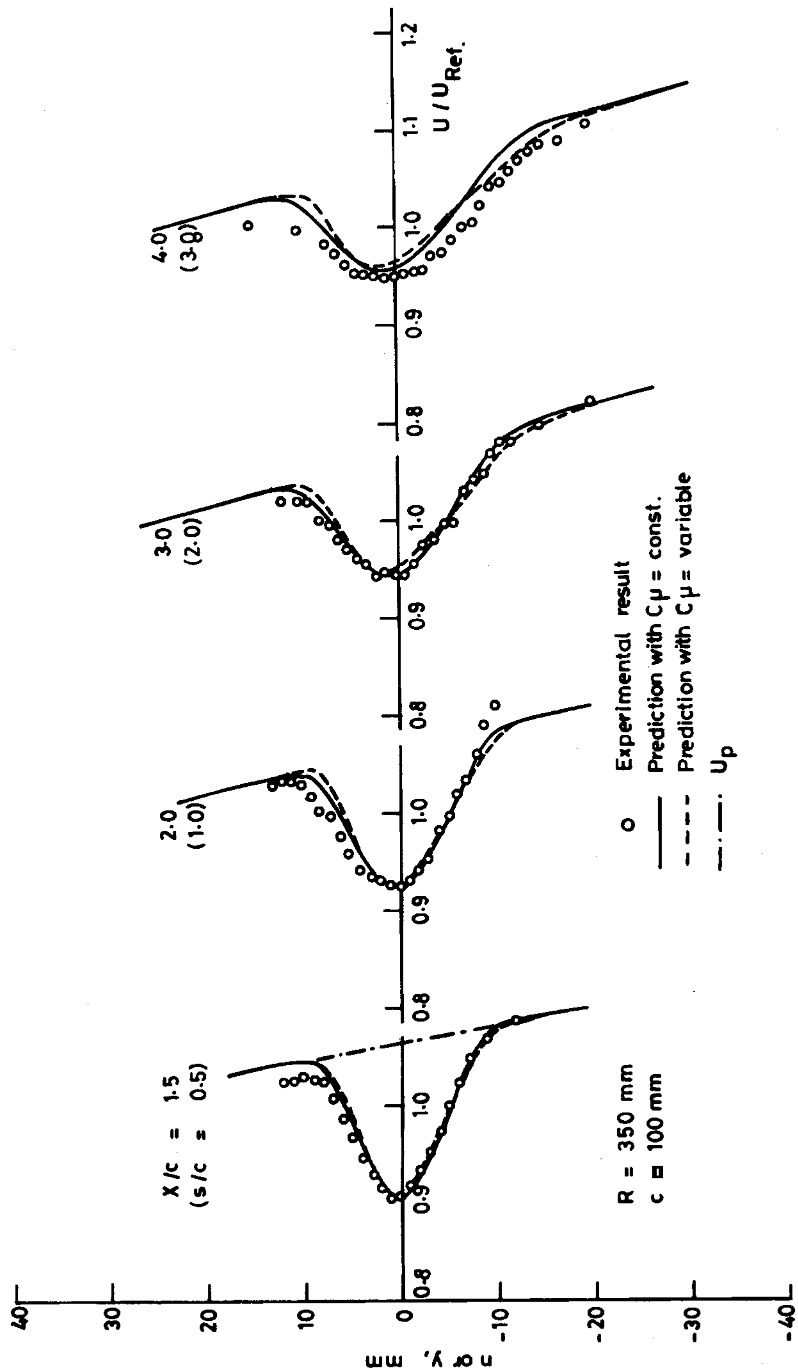

Figure 2. Mean velocity profiles based on reference velocity

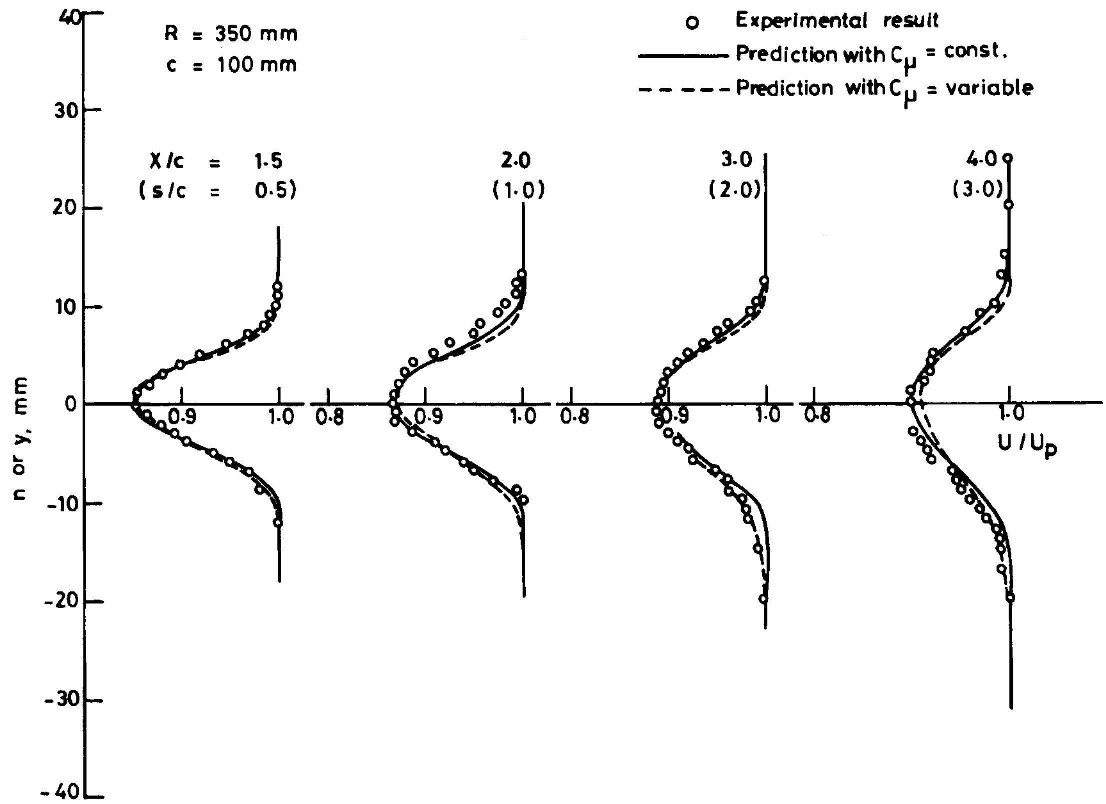

Figure 3. Mean velocity profiles based on potential velocity

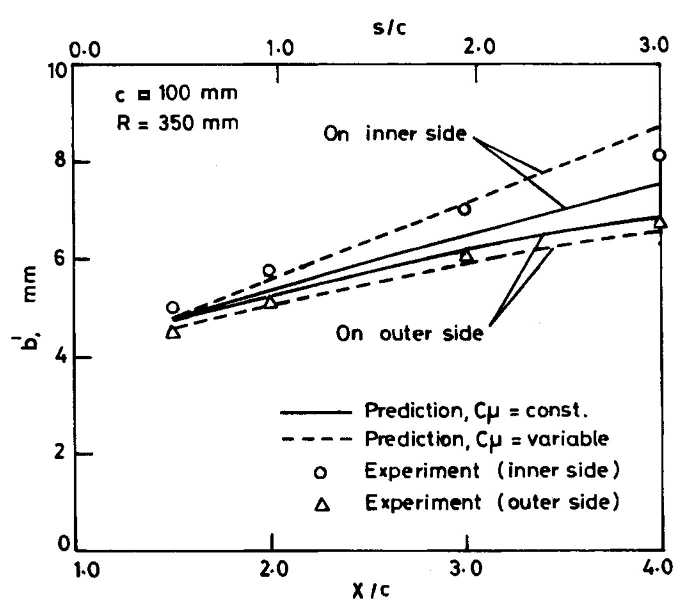

Figure 4. Variation of wake half-width

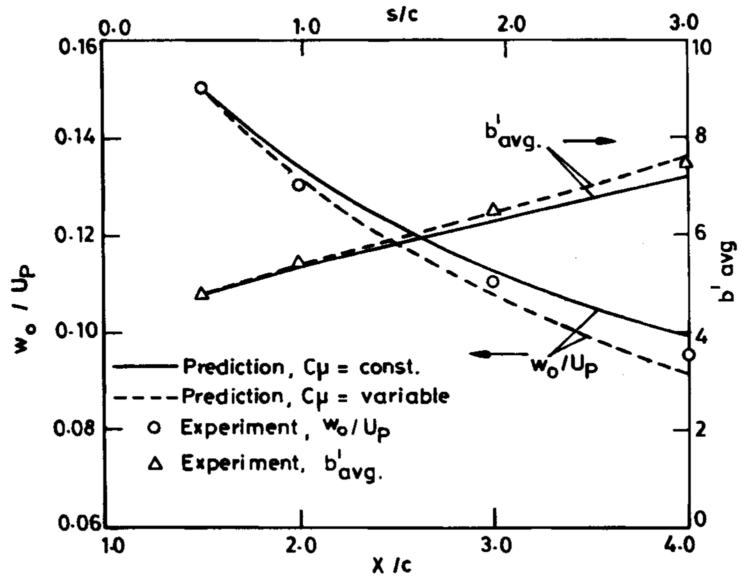

Figure 5. Variation of maximum wake defect and average wake half-width

and Scheuerer[15] and Nakayama,[10] the equations of motion for thin shear flow can be written as

$$\partial U/\partial s+(\partial/\partial n)[(1+n/R)V]=0, \tag{1}$$

$$U\,\partial U/\partial s+V(1+n/R)\,\partial U/\partial n=-UV/R-(1/\rho)\partial p/\partial s-2\overline{u'v'}/R+(1+n/R)\,\partial\overline{u'v'}/\partial n, \tag{2}$$

$$\begin{aligned}U\,\partial k/\partial s+V(1+n/R)\,\partial k/\partial n=&(1+n/R)\,\{\nu_t[\partial U/\partial n-(U/R)/(1+n/R)]^2-\varepsilon\}\\&+(\partial/\partial n)[(\nu_t/\sigma_k)(1+n/R)\,\partial k/\partial n],\end{aligned} \tag{3}$$

$$\begin{aligned}U\,\partial\varepsilon/\partial s+V(1+n/R)\,\partial\varepsilon/\partial n=&(1+n/R)\,\{C_{\varepsilon1}\nu_t(\varepsilon/k)[\partial U/\partial n-(U/R)/(1+n/R)]^2-C_{\varepsilon2}\varepsilon^2/k\}\\&+(\partial/\partial n)[(\nu_t/\sigma_\varepsilon)(1+n/R)\,\partial\varepsilon/\partial n],\end{aligned} \tag{4}$$

where $\overline{u'v'}=\nu_t[\partial U/\partial n-(U/R)/(1+n/R)]$, $\nu_t=C_\mu k^2/\varepsilon$, $C_\mu=0{\cdot}09$, $\sigma_k=1{\cdot}0$, $C_{\varepsilon1}=1{\cdot}44$, $C_{\varepsilon2}=1{\cdot}92$, $\sigma_\varepsilon=1{\cdot}3$, U and V are the mean velocities in the s- and n-directions (Figure 1(b)) respectively, u' and v' are the fluctuating velocity components of U and V respectively, k is the turbulent kinetic energy and ε is the rate of dissipation of k. The s-co-ordinate line coincides with the centreline of the duct and n is normal to it. C_μ, σ_k, $C_{\varepsilon1}$, $C_{\varepsilon2}$ and σ_ε are the constants in the k–ε model.

The above set of equations becomes parabolic in nature when $\partial p/\partial s$ is prescribed. In a straight wake, there is no cross-stream pressure gradient and $\partial p/\partial s$, a constant across the wake, is obtained by applying Bernoulli's equation to the flow outside the wake. In the present case, we take advantage of the derivation done by Nakayama,[10] which shows that when the shear layer is thin compared to the radius R, then the pressure p is given by

$$p/\rho+\tfrac{1}{2}U_p^2=p_{\mathrm{Ref}}/\rho+\tfrac{1}{2}U_{\mathrm{Ref}}^2=\text{const}, \tag{5}$$

where $U_p(s,n)$ is the potential flow velocity obtained by joining the potential flow velocity distributions on the upper and lower sides of the wake. It was found in the experimental

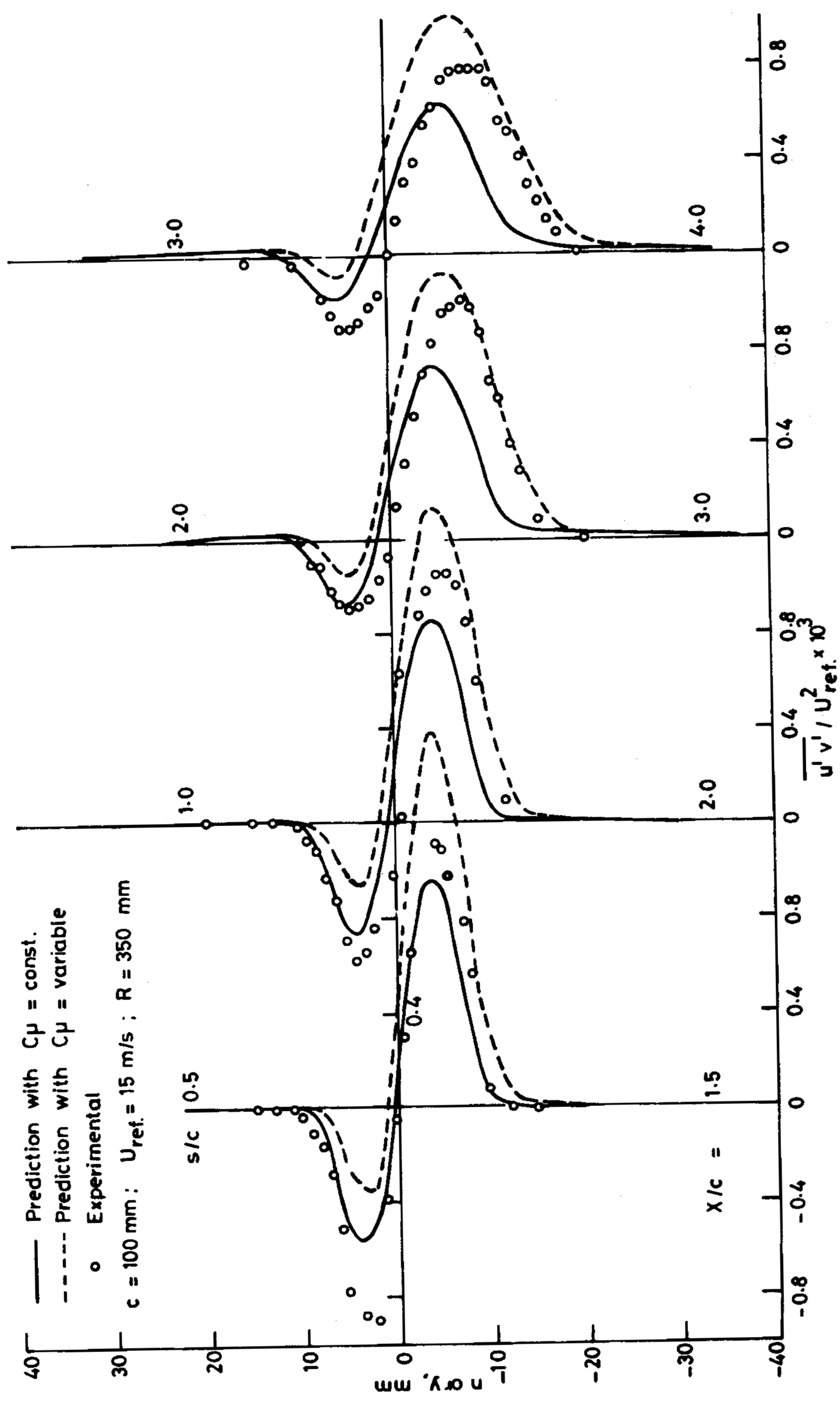

Figure 6. Variation of Reynolds shear stress

investigation of Ramjee *et al.*[11] that the velocity distributions outside the wake are almost same with and without the aerofoil and follow a straight line. Hence we use the experimental distribution of U_p in the duct without the aerofoil and calculate $\partial p/\partial s$ using equation (5), i.e.

$$-(1/\rho)\,\partial p/\partial s = U_p \partial U_p/\partial s. \tag{6}$$

It may be remarked that So and Mellor[16] also used a similar procedure to estimate the potential velocity needed to calculate the momentum and displacement thickness of a boundary layer developing on a curved surface. In the present investigation U_p can be expressed as

$$U_p = U_{pc}(s) - C(s)\,n, \tag{7}$$

where $U_{pc}(s)$ is the potential flow velocity at the centreline of the duct and $C(s)$ is the slope of the straight line portion of the U_p versus n curve at a given location. U_{pc} was found to be almost constant along the duct with a value of $1{\cdot}065U_{\mathrm{Ref}}$. The values of the slope C were calculated from velocity profiles at measured stations. A cubic polynomial curve was used to get a smooth fit to these values.

After $\partial p/\partial s$ is known, a numerical solution to equations (2)–(4) is obtained by prescribing the profiles of U, k and ε at a suitable starting station and using the finite volume scheme of Patankar and Spalding.[17] The V-component of velocity is obtained by taking $V=0$ at the duct centreline and then integrating the continuity equation. Details of the discretization, the solution of the resulting equations, etc. are given in Reference 18.

As mentioned earlier, the first station for which the experimental results are available is at $1{\cdot}5c$ (or at $s/c=0{\cdot}5$) behind the aerofoil. Hence it was decided that (i) the calculations should begin ahead of this station and (ii) the values of w_0 and b'_{avg} at the starting station used to generate the initial profiles should be such that the calculated velocity profile, b' and w_0 are almost the same as the experimental data at $s/c=0{\cdot}5$. This approach provides a reasonable basis for comparing the subsequent developments of experimental and calculated profiles.

The calculation begins at $s/c=0{\cdot}0$. Using guessed values of b'_{avg} and w_0, the velocity profile is generated from the relations[19]

$$F = (U_p - U)/w_0 = (1-\eta^{3/2})^2,$$
$$\eta = n/\delta, \qquad \delta = 2{\cdot}2676\,b'_{\mathrm{avg}}. \tag{8}$$

Since the variation of U_p with n is known at the starting station, U can be worked out using equation (8). Knowing U as a function of n, the distribution of k is calculated as

$$k = (l\,\partial U/\partial n)^2/0{\cdot}3, \qquad l = 0{\cdot}11\,\delta. \tag{9}$$

It may be remarked that in the central portion of the wake $\partial U/\partial n$ is small and becomes zero at the centre. However, k in the central portion of the wake is not much smaller than its maximum value.[3] Hence k is evaluated starting from the edges of the wake and using equation (9) until its maximum value is attained. In the central region k is kept constant at its maximum value.

The initial profile of ε is obtained from

$$\varepsilon = (0{\cdot}3\,k)^{3/2}/l. \tag{10}$$

The profile of V is obtained by taking $V=0$ at the centreline and integrating the continuity equation.

As mentioned earlier, the guessed values of b'_{avg} and w_0 are slightly adjusted so that calculated and experimental values of b'_{avg}, w_0 and velocity profiles are almost same at $x/c=1{\cdot}5$.

As regards the boundary conditions, we need to provide U, k and ε along the upper and lower edges of the wake and V along a line in the shear layer. The distribution of U_p with n is known at each station. Hence the boundary condition on U is that it equals the value of U_p at the upper and lower edges of the calculation domain (U_{pe}). The values of k and ε in the external flow, k_e and ε_e, are calculated using the following relations, which are the forms of equations (3) and (4) in the external flow:

$$U_{pe}\partial k_e/\partial s = -\varepsilon_e, \qquad U_{pe}\partial \varepsilon_e/\partial s = C_{\varepsilon 2}\varepsilon_e^2/k_e. \tag{11}$$

The value of k_e at the initial station is taken equal to $0{\cdot}0001\,U_{pe}^2$ and ε_e is calculated using equation (10). Badrinarayanan *et al.*[20] have shown that even in an unsymmetric wake, V can be taken as zero along the centreline without causing significant error.

A grid with 109 points across the wake as described by Badrinarayanan *et al.*[20] is used and gives grid-independent results.

RESULTS AND DISCUSSION

Figures 2–6 compare the calculated results with the experimental data. The solid lines correspond to the values calculated using the k–ε model with standard constants. The calculated velocity profiles of U/U_{Ref} and U/U_p are in close agreement with experiments (Figures 2 and 3). However, the calculated values of b'_{avg} are lower and those of w_0 are higher. The calculated and experimental distributions of $\overline{u'v'}$ are similar, but the peak values of the calculated profiles are lower, especially on the inner side. However, the rapid decrease in the value of $\overline{u'v'}$ is not due to relaminarization but due to changes in the velocity profiles. From Figure 2 it is seen that as x increases, the velocity defect decreases. At the same time the velocity is lower on the outer side of the wake. These two factors combine to decrease the velocity gradient $\partial U/\partial n$ on the outer side and hence reduce the shear stress.

To examine whether the agreement between calculated and experimental values can be improved, trials were done with C_μ dependent on the local curvature. Leschziner and Rodi[21] have suggested the following expression for C_μ to account for the effect of curvature:

$$C_\mu = \max\{0{\cdot}025,\ 0{\cdot}09/[1+0{\cdot}57(k^2/\varepsilon^2)(\partial U/\partial s + U/R)\,U/R]\}. \tag{12}$$

Results of calculations using this modification are also shown in Figures 2–6. They are denoted by dotted lines. It is seen that there is improved agreement between calculated and experimental values on the inner side but slight worsening of the agreement on the outer side. Thus, keeping in view the range of uncertainty in the experimental data, especially in $\overline{u'v'}$, it can be said that the standard k–ε model gives satisfactory results.

CONCLUSIONS

On the basis of experimental investigations on the development of an aerofoil wake in a curved stream and calculations done using the k–ε model of turbulence, the following observations can be made.

(i) The mean velocity profile in the wake is asymmetric; the half-width of the wake is more on the inner side of the curved duct than on the outer side.

(ii) The decrease in the velocity defect with distance along the duct and the lower potential flow velocity on the outer side cause the velocity gradient to be lower on the outer side and consequently the turbulent shear stress decreases rapidly on the outer side in addition to the curvature effect.

(iii) k–ε model of turbulence with standard constants is able to satisfactorily reproduce the above experimentally observed behaviour of the wake. Making the model constant C_μ dependent on the radius of curvature improves the agreement on the inner side but slightly worsens it on the outer side.

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